Square-Root Gauge Theory and Modified Gravity – Gravity-Assisted Confinement/Deconfinement and Emergent Electro-Weak Symmetry Breaking in Cosmology

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In the present contribution to the proceedings of MG17, the main aim is to elucidate the physically important effects of a special nonlinear gauge field with a square-root of the standard Maxwell Lagrangian in its action, interacting with a specific non-canonical modified $f(R) = R + R^2$ gravity formulated in terms of metric-independent spacetime volume elements and in addition coupled to the bosonic fields of the standard electroweak particle model. When applied in the context of cosmological evolution, the above theory consistently describes absence (suppression) of charge confinement and electroweak (Higgs) spontaneous breakdown in the "early" Universe, whereas in the post-inflationary "late" universe primarily the presence of the "square-root" nonlinear gauge field dynamically triggers both the appearance of QCD-like confinement and dynamical generation of the Higgs effect, as well as dynamical generation of non-zero cosmological constant.

Keywords: charge confining square-root nonlinear gauge field system, modified gravity theories, non-Riemannian volume-forms; global Weyl-scale symmetry spontaneous breakdown; flat regions of scalar inflaton potential

1. Introduction

Our principal task in the present contribution is to present a consistent approach from first principles, i.e., from Lagrangian action principle, to describe consistent mechanisms driving the appearance, respectively the suppression, of confinement and electroweak spontaneous symmetry breaking during the various stages in the cosmological evolution of the Universe^{[1](#page-17-0)}-^{[7](#page-17-1)}.

To this effect we recall that the standard model of cosmology is the so called ΛCDM model, which came about after the discovery of the late acceleration of the Universe (for review, see e.g. Ref.^{[8](#page-17-2)}. Here one thinks of the very "early" universe as undergoing an inflationary phase and then asks about what can produce a full history of the universe. A solution for example is the idea of a quintessential scalar field driving the inflation and then the slowly accelerated phase of the universe as was done in Refs.^{[9](#page-17-3)}.

In what follows we will proceed in two steps. First, we will consider a special kind of a nonlinear (Abelian or non-Abelian) gauge field theory with a square-root of the standard Maxwell/Yang-Mills kinetic term which possess the remarkable property of producing charge confinement. In what follows it will be called for short "square-root" gauge field theory.

Then we will discuss in some detail the main interesting properties of a new type of non-canonical modified (extended) gravity-matter theory, in particular, its implications for cosmology. Namely we will consider modified $f(R) = R + R^2$ gravity coupled in a non-standard way to a scalar "inflaton" field, to the bosonic fields (including the Higgs field) of the standard electroweak particle model, as well as to the above mentioned "square-root" nonlinear gauge field which simulates QCD confining dynamics. Thus, in this way our model will represent qualitatively modified gravity coupled to the whole (bosonic part of the) standard model of elementary particle physics.

The most important non-standard feature of the above model is its construction in terms of non-Riemannian spacetime volume-forms (alternative metricindependent generally covariant volume elements) defined in terms of auxiliary an-tisymmetric tensor gauge fields of maximal rank (see Refs.^{[10](#page-17-4)[,11](#page-17-5)} for a consistent geo-metrical formulation, which is an extension of the originally proposed method^{[12](#page-17-6)[,13](#page-18-0)}). The latter auxiliary volume-form gauge fields were shown in Refs.^{[11](#page-17-5)[,14](#page-18-1)[,15](#page-18-2)} to be almost pure-gauge apart from few arbitrary integration constants, which have the meaning of residual discrete time-conserved degrees of freedom. Thus, they do not produce any additional propagating field-theoretic degrees of freedom (see also Sect.3 below).

On the other hand the non-Riemannian spacetime volume-forms trigger a series of important physical features unavailable in ordinary gravity-matter models with the standard Riemannian volume element (given by the square-root of the determinant of the Riemannian metric):

(i) The "inflaton" φ develops a remarkable effective scalar potential in the Einstein frame possessing an infinitely large flat region for large negative φ describing the "early" universe evolution;

(ii) In the absence of the $SU(2) \times U(1)$ iso-doublet (Higgs) scalar field, the "inflaton" effective potential has another infinitely large flat region for large positive φ at much lower energy scale describing the "late" post-inflationary (dark energy dominated) universe;

(iii) Inclusion of the $SU(2) \times U(1)$ iso-doublet scalar field σ – without the usual tachyonic mass and quartic self-interaction term – introduces a drastic change in the total effective scalar potential in the post-inflationary universe: the effective potential as a function of σ dynamically acquires exactly the electroweak Higgs-type spontaneous symmetry breaking form. The latter is a remarkable explicit realization of Bekenstein's idea^{[16](#page-18-3)} for a gravity-assisted dynamical electroweak spontaneous

symmetry breaking.

(iv) Further important features arise because of the coupling to the above mentioned additional nonlinear gauge field whose Lagrangian contains a square-root of the standard Maxwell/Yang-Mills kinetic term. The latter is known to describe charge confinement in flat spacetime^{[17](#page-18-4)} as well as in curved spacetime for static spherically symmetric field configurations (Refs. 11,15 11,15 11,15 11,15 ; see also Eq.[\(21\)](#page-5-0) below). This is a simple implementation of 't Hooft's idea [18](#page-18-5) about confinement being produced due to the presence in the energy density of electrostatic field configurations of a term linear w.r.t. electric displacement field in the infrared region (arising presumably as an appropriate infrared counterterm). Therefore, the addition of the "square-root" nonlinear gauge field will simulate the strong interactions QCD-like dynamics.

As a result, in the Einstein frame of the present modified (extended) gravity+matter model the above outlined formalism allows us to achieve:

(a) Bekenstein-inspired^{[16](#page-18-3)} gravity-inflaton-assisted dynamical generation of Higgs-type electroweak spontaneous symmetry breaking in the "late" universe, while there is no electroweak breaking in the "early" universe;

(b) Simultaneously we obtain gravity-inflaton-assisted dynamical generation of charge confinement in the "late" universe as well as gravity-suppression of confinement, i.e., deconfinement in the "early" universe.

Finally, in the last Section we will briegly describe a closely related model of modified gravity interacting with a nonlinear "square-root" gauge field which de-scribes gravitational bags resembling the solitonic "constituent quark" model^{[19](#page-18-6)} and MIT bags in QCD phenomenology^{[20](#page-18-7)[,21](#page-18-8)}.

2. Charge-Confining Nonlinear Gauge Field

We start by first exhibiting the charge-confining feature of a special nonlinear gauge field theory whose Lagrangian action contains an additional term being a squareroot of the usual Maxwell Lagrangian. To this end we will follow the steps of the derivation in Ref. 17 of effective "Cornell"-type confining potential^{[22](#page-18-9)[–24](#page-18-10)} be-tween quantized charged fermions based on the general formalism^{[25](#page-18-11)} for quantization within the canonical Hamiltonian approach a'la Dirac of truncated gauge and gravity theories by imposing explicitly spherical symmetry on the pertinent Lagrangian action.

The corresponding nonlinear gauge field action in curved space-time background with metric $g_{\mu\nu}$ coupled to an external (charged matter) current J^{μ} reads:

$$
S = \int d^4x \sqrt{-g} \Big[L(F^2) + A_\mu J^\mu \Big] , \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \tag{1}
$$

with dimensionful coupling constant f_0 of the nonlinear gauge field term and where:

$$
F^2 \equiv F^2(g) = F_{\mu\nu} F_{\kappa\lambda} g^{\mu\kappa} g^{\nu\lambda} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \; . \tag{2}
$$

The action [\(2\)](#page-2-0) yields the following equations of motion:

$$
\partial_{\nu} \left(\sqrt{-g} 4L'(F^2) F^{\mu \nu} \right) + \sqrt{-g} J^{\mu} = 0 \quad , \quad L'(F^2) = -\frac{1}{4} \left(1 - \frac{f_0}{\sqrt{-F^2}} \right) , \tag{3}
$$

whose $\mu = 0$ component – the nonlinear "Gauss law" constraint equation reads:

$$
\frac{1}{\sqrt{-g}} \partial_i (\sqrt{-g} D^i) = J^0 \quad , \quad D^i = \left(1 - \frac{f_0}{\sqrt{-F^2}} \right) F^{0i} , \tag{4}
$$

with $\vec{D} \equiv (D^i)$ denoting the electric displacement field nonlinearly related to the electric field $\vec{E} \equiv (F^{0i})$ as in the last relation [\(4\)](#page-3-0).

The special nonlinear gauge field theory [\(1\)](#page-2-1) possesses a nontrivial vacuum solution $\sqrt{-F_{\text{vac}}^2} = f_0$, which according to the second Eq.[\(4\)](#page-3-0) implies simultaneously: (a) vanishing of the electric displacement field, $\vec{D}=0$ meaning zero observed charge, and at the same time (b) a nontrivial electric field \vec{E} . This can be viewed as the simplest classical manifestation of charge confinement: $\vec{D} = 0$ and nontrivial \vec{E} .

Indeed, for instance in static spherically symmetric fields in a static spherically symmetric space-time metric, e.g. of the form:

$$
ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = -\mathcal{A}(r)dt^2 + \frac{dr^2}{\mathcal{A}(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi\right),
$$
 (5)

with general $\mathcal{A}(r) = -g_{00} = 1/g_{rr}$, the only surviving component of $F_{\mu\nu}$ is the non-vanishing radial component of the electric field $E^r = -F_{0r}$, so that $\sqrt{-F_{\text{vac}}^2} = \sqrt{2|E|} = f$. $\sqrt{2}|\vec{E}| = f_0.$

In order to exhibit the charge-confining feature of the nonlinear gauge theory [\(1\)](#page-2-1) we will employ the canonical Hamiltonian treatment in Ref. 25 and will truncate the non-linear gauge field action to purely spherically symmetric fields, i.e., we will take $F_{0r} = \partial_0 A_r - \partial_r A_0$ independent of the space angles and the rest of the components of $F_{\mu\nu}$ being zero and also will use the specific example of a spherically symmetric de Sitter-type metric (cf. [\(5\)](#page-3-1). The action of the truncated theory reads:

$$
S_{\text{truncated}} = \int dt \int dr 4\pi r^2 \left[\frac{1}{2} F_{0r}^2 - \frac{f_0}{\sqrt{2}} |F_{0r}| + A_0 J^0 + A_r J^r \right]. \tag{6}
$$

Let us note that in [\(6\)](#page-3-2) there is no explicit dependence on the Riemannian metric coefficient $\mathcal{A}(r)$ (cf.[\(5\)](#page-3-1)). It is now straightforward to apply the canonical Hamiltonian quantization procedure to [\(6\)](#page-3-2) within the Dirac formalism for constrained dynamical systems (e.g. Ref.^{[26](#page-18-12)}). Obviously, in the case of de Sitter spacetime the radial coordinate r must be restricted to vary up to the de Sitter horizon radius r_H .

The canonically conjugated momenta w.r.t. A_0 and A_r read:

$$
\Pi^0 = 0 \quad , \quad \Pi^r = 4\pi r^2 \Big(F_{0r} - \frac{f_0}{\sqrt{2}} \Big) \,, \tag{7}
$$

where the first one $\Pi^0 = 0$ is the standard primary Dirac constraint known in any gauge theory of Yang-Mills type. For the density of the canonical Hamiltonian one obtains:

$$
\mathcal{H} = \frac{1}{8\pi r^2} (\Pi^r)^2 + \frac{f_0}{\sqrt{2}} \Pi^r + \pi r^2 f_0^2 - A_r J^r + \Pi^r \partial_r A_0 - J^0 A_0 . \tag{8}
$$

Henceforth, for simplicity we will consider the case with no matter current $J^r = 0$. Time-preservation of the primary constraint $\Pi^0 = 0$, i.e., $\frac{d}{dt} \Pi^0 = \left\{ \Pi^0, \mathcal{H} \right\}$ $_{PB} = 0$ yields the standard secondary Dirac constraint – the "Gauss law" constraint:

$$
\Phi_1(r) \equiv \partial_r \Pi^r + J^0 = 0.
$$
\n(9)

Thus, one has to Dirac-canonically quantize the theory with canonical Hamiltonian:

$$
H = \int dr \left[\frac{1}{8\pi r^2} \left(\Pi^r \right)^2 + \frac{f_0}{\sqrt{2}} \Pi^r + \pi r^2 f_0^2 \right]
$$
 (10)

and with two first-class a'la Dirac constraints $\Phi_{0,1} = 0$ ($\Phi_0 \equiv \Pi^0 = 0$ and $\Phi_1 = 0$) as in [\(9\)](#page-4-0)), which have to be supplemented by two canonically conjugate gaugefixing conditions $\chi_{0,1}$. Since A_0 and its conjugate momentum $\Pi^0 = 0$ do not mix with the rest of the canonical variables they have no impact on the pertinent *Dirac* brackets between A_r and Π^r to be promoted to quantum operator commutators upon quantization. Thus we only need to pick an appropriate gauge fixing condition for the "Gauss law" constraint [\(9\)](#page-4-0), which we can choose in the form:

$$
\chi_1(r) \equiv \int_{C(r)} dz^{\lambda} A_{\lambda}(z) . \tag{11}
$$

Here $\int_{C(r)}$ is path integral along a spacelike geodesic $x^{\lambda} = x^{\lambda}(\xi)$ ending at the spacetime point with radial coordinate r . In particular, for the interior de Sitter region $(r \leq r_H)$ this spacelike geodesic $x^{\lambda}(\xi) = (t(\xi), r(\xi))$ can be taken in the form:

$$
t(\xi) = t = \text{const} \quad , \quad r(\xi) = r_H \sin(\xi/r_H) \quad , \quad 0 \le \xi \le \xi_{\text{fin}} \le r_H \frac{\pi}{2} \quad , \quad r(\xi_{\text{fin}}) = r \tag{12}
$$

where ξ is the de Sitter proper distance parameter, so that:

$$
\chi_1(r) \equiv \int_0^r dz A_r(z) , \quad \left\{ \Phi_1(r), \chi_1(r') \right\}_{PB} = \delta(r - r') . \tag{13}
$$

Note that here and below $\delta(r - r')$ denotes the Dirac delta-function on the half-line (both $r, r' > 0$).

It is now straightforward to calculate the Dirac bracket between the canonically conjugate pair given by:

$$
\left\{ A_r(r), \Pi^r(r') \right\}_{DB} = \left\{ A_r(r), \Pi^r(r') \right\}_{PB}
$$

$$
- \int \int dr'' dr''' \left\{ A_r(r), \Phi_1(r'') \right\}_{PB} \left\{ \Phi_1(r''), \chi_1(r''') \right\}_{PB}^{-1} \left\{ \chi_1(r'''), \Pi^r(r') \right\}_{PB}, \quad (14)
$$

by using the standard Poisson bracket $\{A_r(r), \Pi^r(r')\}$ $_{\text{PB}} = \delta(r - r'),$ which yields:

$$
\left\{ A_r(r), \Pi^r(r') \right\}_{\text{DB}} = 2\delta(r - r') . \tag{15}
$$

Upon canonical quantization [\(15\)](#page-4-1) becomes:

$$
\left[\hat{\Pi}^r(r), \hat{A}_r(r')\right] = 2i\delta(r - r') \quad , \text{ i.e. } \hat{\Pi}^r(r) = -2i\delta/\delta A_r(r) \ . \tag{16}
$$

Now, following Ref. 17 we consider a gauge invariant quantum state of two oppositely charged ($\pm e_0$) fermions located at $r = 0$ and $r = L$, respectively, which is explicitly given by:

$$
|\Phi\rangle \equiv |\bar{\Psi}(L)\Psi(0)\rangle = \bar{\Psi}(L)\exp\left\{ie_0 \int_0^L dz A_r(z)\right\}\Psi(0) |0\rangle . \qquad (17)
$$

The average of the quantized canonical Hamiltonian [\(10\)](#page-4-2) in this state [\(17\)](#page-5-1):

$$
\langle \Phi | \dot{H} | \Phi \rangle \equiv V_{\text{eff}}(L) \tag{18}
$$

can be viewed as effective potential between the quantized fermionic pair generated by the nonlinear gauge field theory containing the "square-root" Maxwell term [\(1\)](#page-2-1).

Using relation [\(16\)](#page-5-2) one calculates:

$$
\left[\hat{\Pi}^r(r), ie_0 \int_0^L dz A_r(z)\right] = 2e_0 \theta(L-r) , \qquad (19)
$$

$$
\left[\left[\left(\widehat{\Pi}^r(r)\right)^2, ie_0 \int_0^L dz A_r(z)\right], ie_0 \int_0^L dz A_r(z)\right] = 8e_0^2 \theta(L-r) ,\qquad (20)
$$

where $\theta(r - r')$ denotes the step-function on the half-line (both $r, r' > 0$). Upon plugging $(19)-(20)$ $(19)-(20)$ into (18) we obtain for the effective potential (18) :

$$
V_{\text{eff}}(L) = -\frac{e_0^2}{2\pi} \frac{1}{L} + e_0 f_0 \sqrt{2} L + (L-\text{independent const}), \qquad (21)
$$

which has precisely the form of the "Cornell" potential 2^{2-24} , *i.e.*, a sum of two pieces – an ordinary Coulomb plus a linear confining term.

In fact, we could equally well take the "square-root" nonlinear gauge field A_u to be non-Abelian. We notice that for static spherically symmetric solutions the non-Abelian model effectively reduces to the abelian one^{[17](#page-18-4)}. Thus, the "squareroot" gauge field will simulate the QCD-like confining dynamics. In this sense the constants f_0 and e_0 in [\(21\)](#page-5-0) will play the role of a confinement-strength coupling constant and of a "color" charge, respectively. It is with this interpretation that we will view the nonlinear gauge field action [\(1\)](#page-2-1) in Section 4.

3. Non-Riemannian Volume-Forms - Basic Properties

Since non-Riemannian volume-forms, or metric-independent space-time volume elements (equivalently, generally-covariant integration measures on the space-time manifold) will play a fundamental role in formulating our specific version of modified (extended) gravitational theory, here we will briefly describe basic features.

Volume-forms (space-time volume elements) are given by nonsingular maximal rank differential forms ω :

$$
\int_{\mathcal{M}} \omega(\ldots) = \int_{\mathcal{M}} dx^{D} \Omega(\ldots) , \quad \omega = \frac{1}{D!} \omega_{\mu_{1} \ldots \mu_{D}} dx^{\mu_{1}} \wedge \ldots \wedge dx^{\mu_{D}} , \qquad (22)
$$

$$
\omega_{\mu_1...\mu_D} = -\varepsilon_{\mu_1...\mu_D} \Omega \ , \ dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} = \varepsilon^{\mu_1...\mu_D} dx^D \ . \tag{23}
$$

Here we are using the following conventions for the alternating symbols $\varepsilon^{\mu_1,...,\mu_D}$ and $\varepsilon_{\mu_1,...,\mu_D}$: $\varepsilon^{01...D-1} = 1$ and $\varepsilon_{01...D-1} = -1$). The volume element (integration measure density) Ω transforms as scalar density under general coordinate reparametrizations.

In standard generally-covariant theories (in D space-time dimensions with Lagranian action $S = \int d^D x \sqrt{-g} \mathcal{L}$ the usual Riemannian spacetime volume-form is defined through the "D-bein" (frame-bundle) canonical one-forms $e^A = e^A_\mu dx^\mu$ $(A = 0, \ldots, D - 1)$:

$$
\omega = e^0 \wedge \ldots \wedge e^{D-1} = \det \|e^A_\mu\| \, dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} \ ,
$$

$$
\longrightarrow \Omega = \det \|e^A_\mu\| \, d^D x = \sqrt{-\det \|g_{\mu\nu}\|} \, d^D x \ . \tag{24}
$$

In fact, in order to define a manifestly generally-invariant action principle there is no a priori any obstacle to employ instead of $\sqrt{-g}$ another alternative non-Riemannian volume element as in $(22)-(23)$ $(22)-(23)$ given by a non-singular exact D-form $\omega = dB$ where:

$$
B = \frac{1}{(D-1)!} B_{\mu_1...\mu_{D-1}} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{-1}}, \qquad (25)
$$

so that the non-Riemannian volume element reads:

$$
\Omega \equiv \Phi(B) = \frac{1}{(D-1)!} \varepsilon^{\mu_1...\mu_D} \partial_{\mu_1} B_{\mu_2...\mu_D} . \tag{26}
$$

Here $B_{\mu_1...\mu_{D-1}}$ is an auxiliary rank $(D-1)$ antisymmetric tensor gauge field. $\Phi(B)$, which is in fact the density of the dual of the rank D field strength $F_{\mu_1...\mu_D}$ = $\frac{1}{(D-1)!}\partial_{[\mu_1}B_{\mu_2...\mu_D]} = -\varepsilon_{\mu_1...\mu_D}\Phi(B)$, similarly transforms as scalar density under general coordinate reparametrizations.

At this point it is crucial to emphazise that the presence of non-Riemannian volume element $\Phi(B)$ in a gravity-matter action $S = \int d^D x \Phi(B) \mathcal{L} + \dots$ does not change the number of $field-theoretic$ degrees of freedom – the latter remains the same as with the standard Riemannian measure $\sqrt{-g}$. This important fact was demon-strated in detail by systematic application (see e.g.^{[15](#page-18-2)}) of the canonical Hamiltonian analysis which reveals that the auxiliary tensor gauge field $B_{\mu_1...\mu_{D-1}}$ is (almost) pure-gauge! This is because the total Lagrangian is only linear w.r.t. B-velocities, so it leads to Hamiltonian constraints a'la Dirac. The only remnant of $B_{\mu_1...\mu_{D-1}}$ is a discrete degree of freedom which appears as integration constant M in the equations of motion w.r.t. $B_{\mu_1...\mu_{D-1}}$ (see subsect. 4.2 below). M is in fact a conserved Dirac constrained canonical momentum conjugated to the "magnetic" B-component $\frac{1}{(D-1)!} \varepsilon^{i_1...i_{D-1}} B_{i_1...i_{D-1}}.$

4. Confining Nonlinear Gauge Field Coupled to Modified Gravity and the Bosonic Sector of the Electroweak Standard Model

Now we proceed to our main task to consider in some detail a specific non-canonical modified gravity theory coupled to the bosonic sector of the electroweak standard model and to the nonlinear "square-root" confining gauge field [\(1\)](#page-2-1), as well as to exhibit the impact of the latter on the process of emergence of some fundamental processes during evolution of the Universe, notably charge confinement and Higgs mechanism of electroweak symmetry breaking.

4.1. Modified $f(R)$ -Gravity Model with Non-Riemannian Spacetime Volume-Forms

We start with the following non-canonical $f(R) = R + R^2$ gravity-matter action constructed in terms of two different non-Riemannian volume-forms, i.e. generally covariant metric-independent volume elements (for simplicity we use units with the Newton constant $G_N = 1/16\pi$:

$$
S = \int d^4x \, \Phi(A) \Big[R + L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} \Big] +
$$

$$
\int d^4x \, \Phi(B) \Big[\epsilon R^2 - \frac{1}{4e^2} F^2 - \frac{1}{4g^2} \mathcal{F}^2(A) - \frac{1}{4g'^2} \mathcal{F}^2(B) + \frac{\Phi(H)}{\sqrt{-g}} \Big].
$$
 (27)

Here the following notations are used:

• $\Phi(A)$ and $\Phi(B)$ are two independent non-Riemannian volume-forms given in terms of the dual field-strengths of two auxiliary rank 3 antisymmetric tensor gauge fields $A_{\nu\kappa\lambda}$ and $B_{\nu\kappa\lambda}$.

$$
\Phi(A) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_{\mu} A_{\nu\kappa\lambda} \quad , \quad \Phi(B) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_{\mu} B_{\nu\kappa\lambda} \; . \tag{28}
$$

• $\Phi(H)$ is the dual field-strength of an additional auxiliary tensor gauge field $H_{\nu\kappa\lambda}$, whose presence is crucial for the consistency of the model [\(27\)](#page-7-0):

$$
\Phi(H) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_{\mu} H_{\nu\kappa\lambda} . \tag{29}
$$

- Let us particularly emphasize that we start within the first-order Pala*tini formalism* for the scalar curvature R and the Ricci tensor $R_{\mu\nu}$: $R =$ $g^{\mu\nu}R_{\mu\nu}(\Gamma)$, where $g_{\mu\nu}$, $\Gamma^{\lambda}_{\mu\nu}$ – the metric and affine connection are apriori independent.
- $L_1(\varphi, X)$ is the scalar "inflaton" Lagrangian:

$$
L_1(\varphi, X) = X - f_1 e^{-\alpha \varphi} \ , \ X \equiv -\frac{1}{2} g^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi \ , \tag{30}
$$

where α , f_1 are dimensionful positive parameters.

• $\sigma \equiv (\sigma_a)$ is a complex $SU(2) \times U(1)$ iso-doublet Higgs-like scalar field with Lagrangian:

$$
L_2(\sigma, Y) = Y - m_0^2 \sigma_a^* \sigma_a , \quad Y \equiv -g^{\mu\nu} (\nabla_\mu \sigma)_a^* \nabla_\nu \sigma_a , \tag{31}
$$

where the gauge-covariant derivative acting on σ reads:

$$
\nabla_{\mu}\sigma = \left(\partial_{\mu} - \frac{i}{2}\tau_A\mathcal{A}^A_{\mu} - \frac{i}{2}\mathcal{B}_{\mu}\right)\sigma\,,\tag{32}
$$

with $\frac{1}{2}\tau_A$ (τ_A – Pauli matrices, $A = 1, 2, 3$) indicating the $SU(2)$ generators and \mathcal{A}^A_μ $(A = 1, 2, 3)$ and \mathcal{B}_μ denoting the corresponding electroweak $SU(2)$ and $U(1)$ gauge fields.

• The electroweak gauge field kinetic terms are of the standard Yang-Mills form (all $SU(2)$ indices $A, B, C = (1, 2, 3)$):

$$
\mathcal{F}^{2}(\mathcal{A}) \equiv \mathcal{F}^{A}_{\mu\nu}(\mathcal{A}) \mathcal{F}^{A}_{\kappa\lambda}(\mathcal{A}) g^{\mu\kappa} g^{\nu\lambda} , \quad \mathcal{F}^{2}(\mathcal{B}) \equiv \mathcal{F}_{\mu\nu}(\mathcal{B}) \mathcal{F}_{\kappa\lambda}(\mathcal{B}) g^{\mu\kappa} g^{\nu\lambda} ,
$$
\n
$$
\mathcal{F}^{A}_{\mu\nu}(\mathcal{A}) = \partial_{\mu} \mathcal{A}^{A}_{\nu} - \partial_{\nu} \mathcal{A}^{A}_{\mu} + \epsilon^{ABC} \mathcal{A}^{B}_{\mu} \mathcal{A}^{C}_{\nu} , \quad \mathcal{F}_{\mu\nu}(\mathcal{B}) = \partial_{\mu} \mathcal{B}_{\nu} - \partial_{\nu} \mathcal{B}_{\mu} .
$$
\n(34)

• Finally, we stress on the additional coupling in the action [\(27\)](#page-7-0) to the "square-root" nonlinear (Abelian) gauge field A_u as in [\(1\)](#page-2-1). Since as discussed in the presious Section the latter simulates QCD-like confinement dynamics, we can view [\(27\)](#page-7-0) as a theory describing modified gravitation interacting with the fields of the whole standard model of elementary particles.

Let us note that the structure of action [\(27\)](#page-7-0) is uniquely fixed by the requirement for invariance (with the exception of the regular mass term of the iso-doublet Higgslike scalar σ_a under the following global Weyl-scale transformations:

$$
g_{\mu\nu} \to \lambda g_{\mu\nu} , \ \varphi \to \varphi + \frac{1}{\alpha} \ln \lambda , \ A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} , B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} , \quad (35)
$$

$$
\Gamma^{\mu}_{\nu\lambda} , H_{\mu\nu\kappa} , \sigma_a , A_{\mu} , A^A_{\mu} , B_{\mu} - \text{inert} .
$$

In fact, as shown in 15 one can, instead with the usual Higgs-like mass term, start by temporarily introducing an additional auxiliary scalar field ψ with a fully Weyl-scale invariant action $\int d^4x \left[\Phi(A) \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right) - \psi^2 \sigma_a^* \sigma_a \right]$ - $\int d^4x \Phi(C) \psi^2$, where $\Phi(C)$ is another auxiliary (pure-gauge) metric-independent volume element of the same form as in [\(28\)](#page-7-1). Equations of motion w.r.t. $C_{\nu\kappa\lambda}$ imply on-shell $\psi = m_0 \equiv$ const, thus recovering [\(27\)](#page-7-0).

It is also very important to stress that, as demonstrated in Ref.^{[15](#page-18-2)} where systematic canonical Hamiltonian treatment of gravity-matter theories with non-Riemannian volume-forms has been worked out, all auxiliary tensor gauge fields $A_{\mu\nu\kappa}, B_{\mu\nu\kappa}, H_{\mu\nu\kappa}$ are almost pure-gauge up to few residual discrete degrees of freedom (the dynamically energing integration constants M_1, M_2, χ_2 in the next Subsection).

4.2. Derivation of the Einstein-Frame Action. Effective Scalar Field Potential

To this end now we first consider the solutions of the equations of motion of the initial action [\(27\)](#page-7-0) which includes the square root term for the gauge fields, and an electroweak structure similar to that of the standard model w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$. which acquire the form of the following algebraic constraints:

$$
R + L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} = -M_1 = \text{const},\qquad(36)
$$

$$
\epsilon R^2 - \frac{1}{4e^2} F^2 - \frac{1}{4g^2} \mathcal{F}^2(\mathcal{A}) - \frac{1}{4g'^2} \mathcal{F}^2(\mathcal{B}) + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const},\qquad(37)
$$

$$
\frac{\Phi(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} , \qquad (38)
$$

where M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless *integra*tion constants. The algebraic constraint Eqs.[\(36\)](#page-9-0)-[\(38\)](#page-9-0) are the Lagrangian-formalism counterparts of the Dirac first-class Hamiltonian constraints on the auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$, $H_{\mu\nu\lambda}$ as discussed in ^{[15](#page-18-2)} and also in ^{[11](#page-17-5)[,14](#page-18-1)}.

The equations of motion of [\(27\)](#page-7-0) w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$ (recall – we are using Palatini formalism):

$$
\int d^4 x \sqrt{-g} g^{\mu\nu} \left(\frac{\Phi_1}{\sqrt{-g}} + 2\epsilon \frac{\Phi_2}{\sqrt{-g}} R \right) \left(\nabla_\kappa \delta \Gamma^\kappa_{\mu\nu} - \nabla_\mu \delta \Gamma^\kappa_{\kappa\nu} \right) = 0 \tag{39}
$$

yield a solution for $\Gamma^{\mu}_{\nu\lambda}$ as a Levi-Civita connection:

$$
\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa} \left(\partial_{\nu}\bar{g}_{\lambda\kappa} + \partial_{\lambda}\bar{g}_{\nu\kappa} - \partial_{\kappa}\bar{g}_{\nu\lambda}\right) , \qquad (40)
$$

w.r.t. to the following Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$
\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R)g_{\mu\nu} \quad , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \quad , \tag{41}
$$

 χ_2 as in [\(38\)](#page-9-0). Upon using relation [\(36\)](#page-9-0) and notation (38) Eq.[\(41\)](#page-9-1) can be written as:

$$
\bar{g}_{\mu\nu} = \left[\chi_1 - 2\epsilon \chi_2 \Big(L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} + M_1 \Big) \right] g_{\mu\nu} . \tag{42}
$$

Varying [\(27\)](#page-7-0) w.r.t. the original metric $g_{\mu\nu}$ and using relations [\(36\)](#page-9-0)-[\(38\)](#page-9-0) we have the "pre-Einstein" equations:

$$
\chi_1 \left[R_{\mu\nu} + \frac{1}{2} \left(g_{\mu\nu} L^{(1)} - T_{\mu\nu}^{(1)} \right) \right] - \frac{1}{2} \chi_2 \left[T_{\mu\nu}^{(2)} + g_{\mu\nu} \left(\epsilon R^2 + M_2 \right) - 4 \epsilon R R_{\mu\nu} \right] = 0 \ , \ (43)
$$

with χ_1 and χ_2 as in [\(41\)](#page-9-1) and [\(38\)](#page-9-0), and $T_{\mu\nu}^{(1,2)}$ being the canonical energy-momentum tensors:

$$
T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2 \frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} . \tag{44}
$$

of the initial scalar+gauge field Lagrangians in the original action [\(27\)](#page-7-0):

$$
L^{(1)} = L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} , \quad L^{(2)} = -\frac{1}{4e^2} F^2 - \frac{1}{4g^2} \mathcal{F}^2(\mathcal{A}) - \frac{1}{4g'^2} \mathcal{F}^2(\mathcal{B}) .
$$
\n(45)

Taking the trace of Eqs.[\(43\)](#page-9-2) and using again relation [\(36\)](#page-9-0) we solve for the ratio χ_1 [\(41\)](#page-9-1):

$$
\chi_1 = 2\chi_2 \frac{T^{(2)}/4 + M_2}{L^{(1)} - \frac{1}{2}T^{(1)} - M_1},\tag{46}
$$

where $T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu}$. Explicitly we obtain from [\(46\)](#page-10-0):

$$
\chi_1 = \frac{1}{2\chi_2 M_2} \Big(f_1 e^{-\alpha \varphi} + m_0^2 \sigma^* \sigma - M_1 \Big) \tag{47}
$$

The Weyl-rescaled metric $\bar{g}_{\mu\nu}$ [\(42\)](#page-9-3) can be written explicitly as:

$$
\bar{g}_{\mu\nu} = \chi_1 \tilde{\Omega} g_{\mu\nu} \ \ , \ \ \tilde{\Omega} \equiv \frac{1 + \frac{\epsilon}{M_2} \left(f_1 e^{-\alpha \varphi} + m_0^2 \sigma^* \sigma - M_1 \right)^2}{1 + 2\epsilon \chi_2 (\bar{X} + \bar{Y} - \frac{1}{2} f_0 \sqrt{-F^2})} \ , \ \ (48)
$$

$$
\bar{X} \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_{\mu}\varphi\partial_{\nu}\varphi \ , \ \ \bar{Y} \equiv -\bar{g}^{\mu\nu}(\nabla_{\mu}\sigma)_{a}^{*}\nabla_{\nu}\sigma_{a} \ , \ \ \bar{F}^{2} \equiv F_{\mu\nu}F_{\kappa\lambda}\bar{g}^{\mu\kappa}\bar{g}^{\nu\lambda} \ . \tag{49}
$$

Now, we can bring Eqs.[\(43\)](#page-9-2) into the standard form of Einstein equations in the second-order formalism for the Weyl-rescaled metric $\bar{g}_{\mu\nu}$ [\(48\)](#page-10-1), i.e., the *Einstein*frame equations:

$$
R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T_{\mu\nu}^{\text{eff}}
$$
(50)

with effective energy-momentum tensor corresponding according to the definition [\(44\)](#page-9-4):

$$
T_{\mu\nu}^{\text{eff}} = g_{\mu\nu} L_{\text{eff}} - 2 \frac{\partial}{\partial g^{\mu\nu}} L_{\text{eff}} \tag{51}
$$

to the following effective *Einstein-frame* matter Lagrangian (using short-hand no-tations [\(45\)](#page-10-2), and with χ_1 as in [\(47\)](#page-10-3) and Ω as in [\(48\)](#page-10-1)):

$$
L_{\text{eff}} = \frac{1}{\chi_1 \widetilde{\Omega}} \left\{ L^{(1)} + M_1 + \frac{\chi_2}{\chi_1 \widetilde{\Omega}} \left[L^{(2)} + M_2 + \epsilon (L^{(1)} + M_1)^2 \right] \right\}.
$$
 (52)

Thus, for the the full Einstein-fr[a](#page-10-4)me action, $^{\circ}$ where all quantities defined w.r.t. Einstein-frame metric [\(41\)](#page-9-1) are indicated by an upper bar, we obtain:

$$
S = \int d^4x \sqrt{-\bar{g}} \Big[R(\bar{g}) + L_{\text{eff}}(\varphi, \bar{X}; \sigma, \bar{Y}; \bar{F}^2, \bar{\mathcal{F}}(\mathcal{A})^2, \bar{\mathcal{F}}(\mathcal{B})^2) \Big]. \tag{53}
$$

^aIn the absence of the "square-root" gauge field, the structure of the Einstein-Frame Action and the associated effective scalar field potential with applications to quintessential inflationary scenarios was studied in Refs. [27](#page-18-13) .

where after using the expressions for χ_1 [\(47\)](#page-10-3) and Ω (second Eq.[\(48\)](#page-10-1)) the explicit form of L_{eff} [\(52\)](#page-10-5) reads:

$$
L_{\text{eff}} = (\bar{X} + \bar{Y})(1 - 4\epsilon \chi_2 \mathcal{U}(\varphi, \sigma)) + \epsilon \chi_2 (\bar{X} + \bar{Y})^2 (1 - 4\epsilon \chi_2 \mathcal{U}(\varphi, \sigma))
$$

$$
- (\bar{X} + \bar{Y})\sqrt{-\bar{F}^2} \epsilon \chi_2 f_{\text{eff}}(\varphi, \sigma) - \frac{1}{2} f_{\text{eff}}(\varphi, \sigma) \sqrt{-\bar{F}^2}
$$

$$
- \mathcal{U}(\varphi, \sigma) - \frac{1}{4e_{\text{eff}}^2(\varphi, \sigma)} \bar{F}^2 - \frac{\chi_2}{4g^2} \bar{\mathcal{F}}^2 (\mathcal{A}) - \frac{\chi_2}{4g'} \bar{\mathcal{F}}^2 (\mathcal{B}) \tag{54}
$$

Here \bar{X} , \bar{Y} , \bar{F}^2 are as in [\(49\)](#page-10-1) (and similarly for $\bar{\mathcal{F}}{(\mathcal{A})}^2$, $\bar{\mathcal{F}}{(\mathcal{B})}^2$). In [\(54\)](#page-11-0) the following notations are used:

• $\mathcal{U}(\varphi,\sigma)$ is the effective scalar field ("inflaton" + Higgs-like) potential:

$$
\mathcal{U}(\varphi,\sigma) = \frac{\left(f_1 e^{-\alpha \varphi} + m_0^2 \sigma^* \sigma - M_1\right)^2}{4\chi_2 \left[M_2 + \epsilon \left(f_1 e^{-\alpha \varphi} + m_0^2 \sigma^* \sigma - M_1\right)^2\right]} \,. \tag{55}
$$

• $f_{\text{eff}}(\varphi, \sigma)$ is the effective confinement-strength coupling constant:

$$
f_{\text{eff}}(\varphi,\sigma) = f_0 \big(1 - 4\epsilon \chi_2 \mathcal{U}(\varphi,\sigma) \big) ; \tag{56}
$$

• $e_{\text{eff}}^2(\varphi, \sigma)$ is the effective "color" charge squared:

$$
e_{\text{eff}}^2(\varphi,\sigma) = \frac{e^2}{\chi_2} \Big[1 + \epsilon e^2 f_0^2 \big(1 - 4\epsilon \chi_2 \mathcal{U}(\varphi,\sigma) \big) \Big]^{-1} \tag{57}
$$

Note that [\(54\)](#page-11-0) is of quadratic "k-essence" type^{[28](#page-18-14)[–31](#page-18-15)} w.r.t. the "inflaton" φ and the Higgs-like σ fields.

5. Gravity Assisted Confinement/Deconfinement and Emergent Higgs Mechanism during Cosmological Evolution

From the explicit form of L_{eff} [\(54\)](#page-11-0) we find that the nonlinear "confining" gauge field A_{μ} develops a nontrivial vacuum field-strength:

$$
\left. \frac{\partial L_{\text{eff}}}{\partial \bar{F}^2} \right|_{\bar{X}, \bar{Y} = 0} = 0 \tag{58}
$$

explicitly given by:

$$
\sqrt{-\bar{F}^2}_{\text{vac}} = f_{\text{eff}}(\varphi, \sigma) e_{\text{eff}}^2(\varphi, \sigma) \tag{59}
$$

Substituting (59) into (54) we obtain the following total effective scalar field potential (with $\mathcal{U}(\varphi,\sigma)$ as in [\(55\)](#page-11-2)):

$$
\mathcal{U}_{\text{total}}(\varphi,\sigma) = \frac{\mathcal{U}(\varphi,\sigma)(1 - \epsilon e^2 f_0^2) + e^2 f_0^2 / 4\chi_2}{1 + \epsilon e^2 f_0^2 (1 - 4\epsilon \chi_2 \mathcal{U}(\varphi,\sigma))} \ , \qquad (\mathbf{0})
$$

which has few remarkable properties.

First, $\mathcal{U}_{total}(\varphi, \sigma)$ [\(60\)](#page-11-3) possesses two infinitely large flat regions as function of φ when σ is fixed:

(a) (-) flat "inflaton" region for large negative values of φ ,

(b) (+) flat "inflaton" region for large positive values of φ ,

respectively, as graphically depicted on Fig.1 (for $m_0 \sigma^* \sigma \leq M_1$) or Fig.2 (for $m_0 \sigma^* \sigma \geq M_1$).

Fig. 1. Qualitative shape of the total effective scalar potential $U_{\text{total}}(60)$ $U_{\text{total}}(60)$ as function of the "inflaton" φ for fixed Higgs-like σ (when $m_0 \sigma^* \sigma \leq M_1$).

Fig. 2. Qualitative shape of the total effective scalar potential U_{total} [\(60\)](#page-11-3) as function of the "inflaton" φ for fixed Higgs-like σ (when $m_0 \sigma^* \sigma \geq M_1$).

- (i) In the (-) flat "inflaton" region:
	- The effective scalar field potential [\(55\)](#page-11-2) reduces to:

$$
\mathcal{U}(\varphi,\sigma=\text{fixed})\simeq \frac{1}{4\epsilon\chi_2}\% \quad \longrightarrow
$$

leading for the total scalar potential [\(60\)](#page-11-3) to:

$$
\mathcal{U}_{\text{total}} \simeq \mathcal{U}_{\text{total}}^{(-)} = \frac{1}{4\epsilon \chi_2} \,, \tag{61}
$$

which implies that all terms containing φ and σ disappear from the Einstein-frame Lagrangian [\(53\)](#page-10-6), *i.e.*, there is no electroweak spontaneous breakdown in the (-) flat "inflaton" region.

- From [\(56\)](#page-11-4) the first relation [\(61\)](#page-13-0) implies $f_{\text{eff}} = 0$, *i.e.*, there is no confinement in the (-) flat "inflaton" region, meaning gravity-suppression of confinement in this region.
- (ii) In the $(+)$ flat "inflaton" region:
	- The effective scalar field potential [\(55\)](#page-11-2) becomes:

$$
\mathcal{U}(\varphi,\sigma) \simeq \mathcal{U}_{(+)}(\sigma) = \frac{\left(m_0^2 \sigma^* \sigma - M_1\right)^2}{4\chi_2 \left[M_2 + \epsilon \left(m_0^2 \sigma^* \sigma - M_1\right)^2\right]} \,,\tag{62}
$$

and, accordingly the total scalar potential [\(60\)](#page-11-3) reads:

$$
\mathcal{U}_{\text{total}}(\varphi,\sigma) \simeq \mathcal{U}_{\text{total}}^{(+)}(\sigma) = \frac{\mathcal{U}_{(+)}(\sigma)(1 - \epsilon e^2 f_0^2) + e^2 f_0^2 / 4\chi_2}{1 + \epsilon e^2 f_0^2 (1 - 4\epsilon \chi_2 \mathcal{U}_{(+)}(\sigma))}
$$
(63)

producing a dynamically generated nontrivial vacuum for the Higgs-like field:

$$
|\sigma_{\text{vac}}| = \sqrt{M_1}/m_0 \,, \tag{64}
$$

i.e., we obtain "gravity-assisted" electroweak spontaneous breakdown in the $(+)$ flat "inflaton" region.

• At the Higgs vacuum [\(64\)](#page-13-1) we have dynamically generated vacuum energy density, i.e. a dynamically generated cosmological constant $\Lambda_{(+)}$ which arises primarily due to the presence of the "square-root" nonlinear gauge field:

$$
\mathcal{U}_{\text{total}}^{(+)}(\sigma_{\text{vac}}) \equiv 2\Lambda_{(+)} = \epsilon e^2 f_0^2 \Big[4\epsilon \chi_2 \big(1 + \epsilon e^2 f_0^2 \big) \Big]^{-1} . \tag{65}
$$

• The effective confinement-strength coupling constant becomes:

$$
f_{\text{eff}} \simeq f_{(+)} = f_0 \big(1 - 4\epsilon \chi_2 \mathcal{U}_{(+)}(\sigma) \big) > 0 ,
$$
 (66)

threfore we obtain "gravity-assisted" charge confinement in the $(+)$ flat "inflaton" region.

As seen from Fig.1 or Fig.2, the two flat "inflaton" regions of the total scalar field potential [\(60\)](#page-11-3) given by $\mathcal{U}_{total}^{(-)} = \frac{1}{4\epsilon_{\chi_2}}$ [\(61\)](#page-13-0) and $\mathcal{U}_{total}^{(+)}(\sigma_{vac}) \equiv 2\Lambda_{(+)} = \epsilon e^2 f_0^2 \left[4\epsilon_{\chi_2}(1+\right.$ $\epsilon e^2 f_0^2$) $\begin{bmatrix} 1 \end{bmatrix}$ [\(65\)](#page-13-2), respectively, can be identified (cf. e.g. Refs. [9](#page-17-3)) as describing the "early" ("inflationary") and "late" (today's dark energy dominated) epochs in the cosmological evolution of the universe provided we take the following numerical values for the parameters in order to conform to the $PLANCK$ data $32,33$ $32,33$:

$$
\mathcal{U}_{\text{total}}^{(-)} \sim 10^{-8} M_{\text{Pl}}^4 \to \epsilon \chi_2 \sim 10^8 M_{\text{Pl}}^{-4} \text{ , } \Lambda_{(+)} \sim 10^{-122} M_{\text{Pl}}^4 \to \frac{e^2 f_0^2}{\chi_2} \sim 10^{-122} M_{\text{Pl}}^4 \text{ ,}
$$
\n(67)

where $M_{\rm Pl}$ is the Planck mass scale.

From the Higgs v.e.v. $|\sigma_{\text{vac}}| = \sqrt{M_1}/m_0$ and the Higgs mass $\frac{M_1 m_0^2}{4 \chi_2 M_2}$ resulting from the dynamically generated Higgs-like potential $\mathcal{U}^{(+)}_{total}(\sigma)$ [\(63\)](#page-13-3) we find:

$$
m_0 \sim M_{\rm EW} \, , \, M_{1,2} \sim M_{\rm EW}^4 \, , \tag{68}
$$

where $M_{\text{EW}} \sim 10^{-16} M_{\text{Pl}}$ is the electroweak mass scale.

6. "Square-Root" Nonlinear Gauge Field and Gravitational Bags

Here we will very briefly mention another interesting effect produced by the "squareroot" nonlinear gauge field [\(1\)](#page-2-1) when interacting with special modified gravity similar to [\(27\)](#page-7-0) without coupling to the fields of the standard electorweak model (σ , A, B) and with sightly more general scalar (called here "dilaton") field Lagrangian than $L_1(\varphi, X)$ [\(30\)](#page-7-2). The starting action in terms of non-Riemannian volume-elements is:

$$
S = \int d^4x \, \Phi_1(A) \Big[R + L^{(1)} \Big] + \int d^4x \, \Phi_2(B) \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} \Big] \,. \tag{69}
$$

where:

$$
L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_{\mu}\varphi\partial_{\nu}\varphi - f_1 \exp\{-\alpha\varphi\} - \frac{f_0}{2}\sqrt{-F^2} ,\qquad (70)
$$

$$
L^{(2)} = -\frac{b}{2}e^{-\alpha\varphi}g^{\mu\nu}\partial_{\mu}\varphi\partial_{\nu}\varphi + f_2 \exp\{-2\alpha\varphi\} - \frac{1}{4e^2}F^2 \,,\tag{71}
$$

with b a numerical parameter - coupling constant of the non-canonical additional "dilaton" kinetic term and where f_2 is another dimensionaful "dilaton" coupling constant like f_1 .

Following the same line of derivation as with [\(52\)](#page-10-5) we arrive at the following Einstein-frame effective matter Lagrangian:

$$
L_{\text{eff}} = A(\varphi)X + B(\varphi)X^2 - \widetilde{U}_{\text{eff}}(\varphi) - \frac{F^2}{4e_{\text{eff}}^2(\varphi)} - \frac{f_{\text{eff}}(\varphi)}{2} \sqrt{-F^2(\bar{g})} - \epsilon \chi_2 f_0 A(\varphi) X \sqrt{-F^2} ,\tag{72}
$$

with
$$
X \equiv -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi
$$
. The coefficient functions in (72) read:
\n
$$
A(\varphi) = 1 - 4 \widetilde{U}_{\text{eff}}(\varphi) \left[\epsilon \chi_2 - \frac{\chi_2 b e^{-\alpha \varphi}}{2(V(\varphi) - M_1)} \right] , \quad B(\varphi) = \epsilon \chi_2 - 4 \widetilde{U}_{\text{eff}}(\varphi) \left[\epsilon \chi_2 - \frac{\chi_2 b e^{-\alpha \varphi}}{2(V(\varphi) - M_1)} \right]^2 , \tag{73}
$$

whereas the effective scalar field potential reads (the same as (55) for $f_2 = 0$ and modulo the Higgs-field terms):

$$
\widetilde{U}_{\text{eff}}(\varphi) = \frac{\left(f_1 e^{-\alpha \varphi} - M_1\right)^2}{4\chi_2 \left[f_2 e^{-2\alpha \varphi} + M_2 + \epsilon \left(f_1 e^{-\alpha \varphi} - M_1\right)^2\right]} \,,\tag{74}
$$

and the effective coupling constants $f_{\text{eff}}(\varphi)$, $e_{\text{eff}}(\varphi)$ are of the same form as in [\(56\)](#page-11-4), [\(57\)](#page-11-5) with $\mathcal{U}(\varphi, \sigma)$ [\(55\)](#page-11-2) replaced by $\tilde{U}_{\text{eff}}(\varphi)$ [\(74\)](#page-15-0).

In Ref.^{[11](#page-17-5)} the Einstein-frame effective Lagrangian (72) corresponding to model [\(69\)](#page-14-1) was analyzed in detail with the following findings:

(a) As a function of the "dilaton" φ , the effective Lagrangian [\(72\)](#page-14-0) possesses two infinite flat regions similar to Fig.1,2 above: (-) flat region for large negative φ and (+) flat region for large negative φ .

(b) There are two type of "dilaton" vacuums: for $\varphi = \text{const}$ (standard vacuum) and for $X = \text{const}$ ("kinetic" vacuum; here exploiting the quadratic "k-essence" form of (72)).

As a result we get 3 different types of phases:

(i) For the phase corresponding to the standard vacuum ($\varphi = \text{const}$) on both flat (\pm) regions we have confinement since in both (\pm) regions we have non-zero effective confinement-strength coupling constant $f_{\text{eff}}(\varphi)$.

(ii) For the phase corresponding to the "kinetic" vacuum $X = \text{const}$ when φ is on the $(+)$ flat region the total effective confinement-strength coupling constant $f_{\text{eff}}(\varphi)$ vanishes, so there is no confinement.

(iii) For $X = \text{const}$ when φ is on the (-) flat region this once again corresponds to a charge-confining phase due to non-vanishing $f_{\text{eff}}(\varphi)$.

As shown in^{[11](#page-17-5)}, in all three cases the spacetime metric we get is de Sitter or Schwarzschild-de Sitter. Both "kinetic vacuums" (ii) and (iii) can exist only within a finite-volume space region below a de Sitter horizon. Extension to the whole space requires matching geometry of phase (iii) to a static spherically symmetric configuration containing the standard constant "dilaton" vacuum in the outer region beyond the de Sitter horizon The extension for the geometry of phase (ii) requires more complicated matching beyond the de Sitter horizon - the exterior region with a nonstandard Reissner-Nordström-de Sitter geometry carrying an additional constant radial background electric field $34,35$ $34,35$. As a result we obtain two classes of gravitational bag-like configurations with properties, which on one hand partially parallel some of the properties of the solitonic "constituent quark" model^{[19](#page-18-6)} and, on the other hand, partially mimic some of the properties of MIT bags in QCD phenomenology^{[20](#page-18-7)[,21](#page-18-8)}.

7. Conclusions

The main focus in the present contribution is on a special kind of nonlinear gauge field whose Lagrangian contains a square-root of the usual Maxwell term, which is known to describe charge confinement in flat space-time, and study its role and

impacts in curved space-time when interacting with specific modified $f(R) = R + R^2$ gravity coupled to the bosonic fields of the standard electroweak model of elementary particles, thus presumably producing notable implications e.g. on cosmological evolution.

There are two sections with preliminary material. In section 2 we review in some detail the derivation of the confining property of the "square-root" (Abelian or non-Abelian) gauge cornell field on quantized point-like fermions in sphericallysymmetric curved space-time, generating the well-known "Cornell" potential $22-24$ $22-24$ among them. In Section 3 we briefly outline the basics of the formalism of the non-Riemannian (metric-independent) space-time volume elements, which is the corner stone in the construction of broad classes of modified gravitational theories

We then proceed to construct the full action in terms of metric-indepedent volume elements of $f(R) = R + R^2$ gravity coupled scalar "inflaton" field, to the electroweak bosonic fields and to the "square-root" nonlinear gauge field which will mimic QCD-like confining dynamics. The form of the starting action is uniquely fixed by requiring global Weyl-scale invariance. Passing further to the pertinent Einstein-frame action we obtain a nontrivial total effective scalar field potential as well as nontrivial effective "color" charge and confinement-strength coupling constant - all functions of the "inflaton" φ and the Higgs σ fields.

The total effective scalar potential posesses a remarkable property - it has two infinitely large flat regions as function of φ (σ fixed): (-) flat region for large negative φ and (+) flat region for large positive φ (cf. Figs.1,2 above) which are appropriate for description of the evolution of the "early" universe (the (-) flat region) and of the "late" universe (the $(+)$ flat region), accordingly.

Taking into account the contribution of the nontrivial "square-root" gauge field vacuum [\(59\)](#page-11-1) to the total effective scalar potential we find that in the (-) flat region there is no generation of symmetry breaking Higgs potential, as well as the effective (field-dependent) confinement-strength effective coupling constant vanishes. Thus, in the "early" universe both charge confinement and electroweak symmetry breaking are gravity-suppressed. On the other hand, in the $(+)$ flat region both the confinement-strength effective coupling is non-zero, as well as the standard electroweak symmetry-breaking Higgs potential is dynamically generated. On top of this thanks to the nonvanishing confinement-strength coupling of the "square-root" nonlinear gauge field a nontrivial non-zero cosmological constant is dynamically generated responsible for the late-time accelerated expansion of the universe.

In the final Section 6 we briefly touched the interesting phenomenon of dynamical creation of gravitational "bags" by the "square-root" gauge field coupled to modified $f(R) = R + R^2$ gravity plus scalar "dilaton", which resemble the sonstituent quak model and/or the MIT bags in QCD phenomenology.

Finally, let us also mention other noteworthy effects of the nonlinear "squareroot" gauge field when coupled to modified gravity-matter (cf.[\(27\)](#page-7-0) without the electroweak bosonic fields). Namely, as shown in Refs. $34,35$ $34,35$ the latter produces nonstandard black hole solutions with constant vacuum electric field and with "hedgehog"-type spacetime asymptotics, which are shown to obey the first law of black hole thermodynamics; new generalized Levi-Civita-Bertotti-Robinson type"tubelike" space-times; new types of charge-"hiding" and charge-confining "thin-shell" wormhole solutions.

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